If S is the subspace of $\mathbf{R}^3$ containing only the zero vector, what is $S^\perp$? If S is spanned by $(1, 1, 1)$, what is $S^\perp$? If S is spanned by $(2, 0, 0)$ and $(0, 0, 3)$, what is $S^\perp$?

1) a) $\langle 0,0,0 \rangle$

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$S^\perp \mathbb{R}^3$

b) $$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$S^\perp$ $x+y+z=0$

c) $$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x=0$

$z=0 \Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} a$

$S^\perp$

Suppose S is spanned by the vectors $(1, 2, 2, 3)$ and $(1, 3, 3, 2)$. Find two vectors that span $S^\perp$. This is the same as solving $A\mathbf{x} = \mathbf{0}$ for which A?

$$\begin{bmatrix} 1 & 2 & 2 & 3 \\ 1 & 3 & 3 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = 0$$

let $c=1$
$d=0$

$a+2b+2c+3d=0$
$-\ a+3b+3c+2d=0$
$-b-c+d=0$

$a-2+2=0$
$\boxed{a=0}$

$-b-1=0$
$\boxed{b=-1}$

let $c=0$
$d=1$

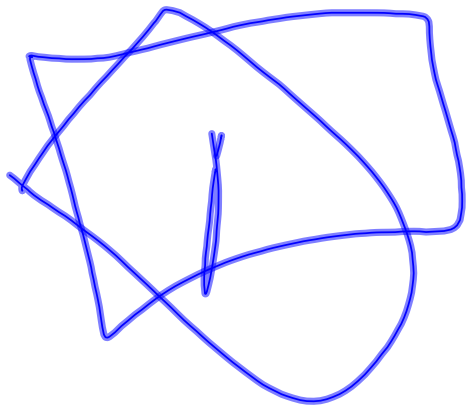
$-b+1=0$
$\boxed{b=1}$

$a+2+3=0$
$\boxed{a=-5}$

$$\begin{bmatrix} -5 \\ 1 \\ 0 \\ 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} y$$

True or false?

$(1, 1, 1)$ is perpendicular to $(1, 1, -2)$ so the planes $x + y + z = 0$ and $x + y - 2z = 0$ are orthogonal subspaces.



Let $\langle a, b, c \rangle$ be a common vector

$$\langle a, b, c] \begin{bmatrix} a \\ b \\ c \end{bmatrix} \neq 0$$

For the given matrix, find the orthogonal complement of

a) column space → left nullspace

b) row space → nullspace

$$\begin{bmatrix} 3 & 1 & 2 \\ 5 & -2 & 1 \\ 1 & -4 & 3 \end{bmatrix}$$

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Project the vector $\boldsymbol{b}$ onto the line through $\boldsymbol{a}$. Check that $\boldsymbol{e}$ is perpendicular to $\boldsymbol{a}$:

(a) $\boldsymbol{b} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ and $\boldsymbol{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (b) $\boldsymbol{b} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ and $\boldsymbol{a} = \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}$.

$$\frac{\begin{bmatrix} 1, & 1, & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}}{\begin{bmatrix} 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$\frac{5}{3\ \cancel{9}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$a^Tb = b^Ta$

$a_1b_1 + a_2b_2 + \cdots$

$= b_1a_1 + b_2a_2$